206. Proposed by **W. J. GREENSTREET, A. M.,** Editor of The Mathematical Gazette, Stroud, England.

$ABCD$ is circumscribed by a circle center O, and it circumscribes a circle radius r. The perpendiculars from C on the sides are x, y, z, u. Show that $\frac{1}{2}AC.BD=r\Sigma x$.

Solution by **G. B. M. ZERR, A. M., Ph. D.,** Parsons, W. Va.

The problem should read "the perpendiculars from O" instead of "the perpendiculars from C.

Let a, b, c, d denote the sides AB, BC, CD, DA, respectively; x the perpendicular on a; y, on b; z, on c; u, on d; R=circum-radius.

Then $x=\sqrt{(R^2-\frac{1}{4}a^2)}$. Now $R=AC/2\sin B$.

$AC^2=a^2+b^2-2ab\cos B=c^2+d^2+2cd\cos B$.

$$\therefore x=\frac{1}{2\sin B}\sqrt{(AC^2-a^2\sin^2 B)}=\frac{b-a\cos B}{2\sin B},\quad y=\frac{a-b\cos B}{2\sin B},$$

$$z=\frac{d+c\cos B}{2\sin B},\quad u=\frac{c+d\cos B}{2\sin B}.\quad \therefore r\Sigma x=\frac{r(a+b+c+d)-r(a+b-c-d)\cos B}{2\sin B},$$

$$r=\frac{2\sqrt{(abcd)}}{a+b+c+d},\quad a+c=b+d,\quad \cos B=\frac{a^2+b^2-c^2-d^2}{2(ab+cd)}.$$

$$\therefore \cos B=\frac{(a-c)(a+c)+(b-d)(b-d)}{2(ab+cd)}=\frac{(a+c)(a+b-c-d)}{2(ab+cd)}.$$

$$\therefore r\Sigma x=\frac{\sqrt{(abcd)}}{\sin B}-\frac{\sqrt{(abcd)}.\,(a+b-c-d)^2}{4(ab+cd)\sin B},\quad \sin B=\frac{2\sqrt{(abcd)}}{ab+cd},$$

$4(ab+cd)\sin B=8\sqrt{(abcd)}$.

$$\therefore r\Sigma x=\frac{ab+cd}{2}-\frac{(a+b-c-d)^2}{8}.\text{ But } d=a+c-d.$$

$$\therefore r\Sigma x=\frac{ab+c(a+c-b)}{2}-\frac{(b-c)^2}{2}=\frac{ab+ac+bc-b^2}{2}$$

$$=\frac{ac+b(a+c)-b^2}{2}=\frac{ac+bd}{2}=\tfrac{1}{2}AC.BD.$$

207. Proposed by **W. W. HART,** University High School, Chicago, Ill.

According to Gauss the circumference of a circle can be divided into n equal parts by ruler and compass only, when n is a prime of the form $2^{2^p}+1$.

The following construction gives good partial results for n equals *any* integer. If AB is the diameter of the circle, and C is the vertex of the equilateral triangle ABC, and if D is a point on AB at the distance $2AB/n$ from A, then draw the line CD cutting the circle at E and F; E being the more remote from